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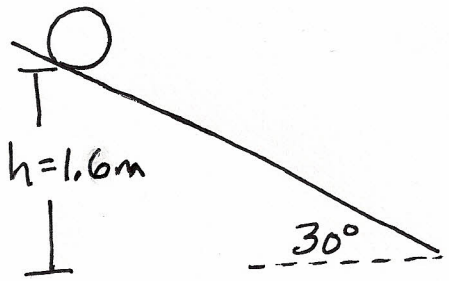
Rotational Motion Practice Problems



A hoop & a disk accelerate w/o slipping from rest down an incline of 30°. What is the speed of each at the bottom? The mass of each is 2kg and the radius is 0.25m.

Cons. of energy

$$\Sigma E_0 = \Sigma E$$

$$U = K + K_{rot}$$

Remember $I_{HOOP} = mr^2$

$\omega = \frac{v}{r}$ ← angular speed

HOOP

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\left(\frac{v}{r}\right)^2$$

mass & radius both cancel out

$$\therefore \quad gh = \frac{1}{2}v^2 + \frac{1}{2}v^2$$

$$gh = v^2$$

$$v = \sqrt{gh} = \sqrt{\left(10\tfrac{m}{s^2}\right)(1.6m)} = 4\,m/s$$

DISK

$$U = K + K_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

The only difference is $I_{DISK} = \frac{1}{2}mr^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

Again mass & radius cancel

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2$$

Notice the Disk has less rotational inertia so it rolls faster.

$$v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}\left(10\tfrac{m}{s^2}\right)(1.6m)} = 4.6\,m/s$$